GCE AS/A level

0978/01

MATHEMATICS FP2
Further Pure Mathematics

P.M. MONDAY, 25 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

CJ*(S12-0978-01)

1. The piecewise function f is defined by

$$f(x) = ax^2 - 8 \quad (x \leqslant 2),$$
$$f(x) = x^3 - bx \quad (x > 2),$$

where a and b are constants.

Given that f and its derivative f' are continuous when $x = 2$, find the values of a and b. [5]

2. Using the substitution $u = e^x$, evaluate the integral

$$\int_0^1 \frac{1}{\left(e^x + 4e^{-x}\right)} dx.$$

Give your answer correct to three decimal places. [6]

3. By putting $t = \tan\left(\frac{x}{2}\right)$, find the general solution to the equation

$$3\sin x = \tan\left(\frac{x}{2}\right).$$ [8]

4. The function f is given by

$$f(x) = \frac{3x^2 - 4x + 1}{(x-2)(x^2+1)}.$$

(a) Express $f(x)$ in partial fractions. [4]

(b) Hence evaluate

$$\int_3^4 f(x)dx,$$

giving your answer in the form $\ln\left(\frac{a}{b}\right)$, where a, b are positive integers. [5]

5. *(a)* The function f is defined by

$$f(x) = x^2 \sin x.$$

Determine whether f is an even function or an odd function. [3]

(b) The function g is defined by

$$g(x) = x^n \sin x,$$

where n is a positive integer. Determine the set of values of n for which g is

(i) an even function,
(ii) an odd function. [3]

6. The function f is defined by

$$f(x) = \frac{2}{x-3} + x - 6.$$

(a) Determine the coordinates of the points where the graph of f intersects the coordinate axes. [5]

(b) Find the coordinates of the stationary points on the graph of f. [5]

(c) State the equation of each of the asymptotes on the graph of f. [2]

(d) Sketch the graph of f. [2]

7. A parabola has equation

$$y^2 - 2y - 8x + 25 = 0.$$

(a) Find

(i) the coordinates of the vertex,
(ii) the coordinates of the focus,
(iii) the equation of the directrix. [6]

(b) The line $y = mx$ cuts the parabola at the points P_1 and P_2.

(i) Obtain a quadratic equation whose roots are the x-coordinates of P_1 and P_2.
(ii) Hence find the gradients of the two tangents from the origin to the parabola. [7]

8. *(a)* Using mathematical induction, prove that

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

for positive integral values of n. [7]

(b) (i) The complex number w is a cube root of the complex number z. Show that

$w\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$ is another cube root of z.

(ii) Write down the real cube root of –8. Using the result in (i), or otherwise, find the two complex cube roots of –8, giving your answers in the form $x + \mathrm{i}y$. [7]